Read Landau-Lifshitz up to p.39 (§15).

(19) Consider the effective potential of the Kepler problem

\[ U_{\text{eff}}(r) = -\frac{\alpha}{r} + \frac{L^2}{2mr^2}. \]

(A) Calculate the value \( r_0 \) where \( U_{\text{eff}}(r) \) has its minimum.

(B) Calculate \( U_{\text{eff}}^{\text{min}} = U_{\text{eff}}(r_0) \).

(C) Assume an energy \( E < 0 \) and calculate the values of the turning points \( r_{\text{min}} \) and \( r_{\text{max}} \).

(D) For which energy range do we have two real solutions?

Due in class, 5 points (2 for C, 1 for A, B, D).

(20) Assume \( 0 < e < 1 \) for the eccentricity and transform the elliptic equation

\[ \frac{p}{r} = 1 + e \cos(\phi), \quad p > 0 \]

into the form \( \frac{x'^2}{a^2} + \frac{y'^2}{b^2} = 1 \).

This means, derive the definitions of \( x', y' \), major half-axis \( a \) and minor half-axis \( b \) in terms of \( x, y, p \) and \( e \). Due Wednesday, October 19 before class, 8 points.

(21) Use the inertial frame of §15 of the book and plot the orbits for the four initial conditions of the table in set 4 (best in one figure). Indicate the initial positions. Due October 24 before class, 12 points.

(22) Calculate the turning point \( \theta_{\text{min}} \) and \( \theta_{\text{max}} \) for the spherical pendulum of the midterm for the special case \( mgR = M_2^2/(4mR^2) = E/2 \). Due in class (6 points).

Read Chapter V of Landau-Lifshitz up to §26, p.79.

(23) Derive the results of problem 3 §21 of Landau and Lifshitz (frequency of oscillations). Due October 26 before class (4 points).

(24) Calculate \( a \) and \( \alpha \) of Eq. (22.4 and 1/2) of Landau and Lifshitz in terms of \( a \) and \( \alpha \) of Eq.(22.4) of Landau and Lifshitz. Then, let \( \gamma = \omega + \epsilon \). Taylor expand

\[ x(t) = a \cos(\omega t + \alpha) + \frac{f [\cos(\gamma t + \beta) - \cos(\omega t + \beta)]}{m (\omega^2 - \gamma^2)} \]

to order \( \epsilon \) and take the limit \( \epsilon \to 0 \). Due in class (4 points). Now October 28 before class.

(25) Calculate the normal modes (eigenfrequencies) of the pendulum of problem 8, set 2. Due October 31 before class (4 points).