
1. The Lagrangians are

\[ L = \frac{1}{2} m \left( \frac{d\vec{r}}{dt} \right)^2 - U , \]
\[ L' = \frac{1}{2} m' \left( \frac{d\vec{r}}{dt'} \right)^2 - U' \]

After substituting \( t' = t \sqrt{m'/m} \) in \( L' \) the Lagrangians agree and the same paths are obtained for

\[ t'/t = \sqrt{m'/m} . \]

2. The Lagrangians are

\[ L = \frac{1}{2} m \left( \frac{d\vec{r}}{dt} \right)^2 - U , \]
\[ L' = \frac{1}{2} m \left( \frac{d\vec{r}}{dt'} \right)^2 - U'' \]

Assume \( U' = \alpha U \). After substituting \( t' = t/\sqrt{\alpha} \) we obtain

\[ L' = \alpha L . \]

Therefore the same paths are obtained for

\[ t'/t = \alpha^{-1/2} = \sqrt{U/U'} . \]