Solution for assignment 26:

Double pendulum solution and plot (continuation of 25).

Let us take minors with respect to the first row of the determinant. For the $\omega_+$ frequency the ratio of the two minors is

$$\frac{\triangle_{1+}}{\triangle_{2+}} = \frac{-1 - \sqrt{2}}{2 + \sqrt{2}} = \frac{(-1 - \sqrt{2})(1 - \sqrt{2})}{(2 + \sqrt{2})(1 - \sqrt{2})} = \frac{1}{-\sqrt{2}}$$

and for $\omega_-$ it is

$$\frac{\triangle_{1-}}{\triangle_{2-}} = \frac{-1 + \sqrt{2}}{2 - \sqrt{2}} = \frac{(-1 + \sqrt{2})(1 + \sqrt{2})}{(2 - \sqrt{2})(1 + \sqrt{2})} = \frac{1}{\sqrt{2}}.$$

Therefore, the solutions (real part) can be written

$$\phi_+(t) = A_+ \cos(\omega_+ t) + B_+ \sin(\omega_+ t),$$
$$\phi_-(t) = A_- \cos(\omega_+ t) + B_- \sin(\omega_+ t),$$
$$\phi(t) = \phi_+(t) + \phi_-(t),$$
$$\psi(t) = -\sqrt{2} \phi_+(t) + \sqrt{2} \phi_-(t).$$

The four constants are determined by the four initial value, e.g., $\phi_0$, $\dot{\phi}_0$, $\psi_0$, $\dot{\psi}_0$ at time $t = 0$:

$$\phi_0 = A_+ + A_-,$$
$$\dot{\phi}_0 = \omega_+ B_+ + \omega_- B_-,$$
$$\psi_0 = \sqrt{2} (-A_+ + A_-),$$
$$\dot{\psi}_0 = \sqrt{2} (-\omega_+ B_+ + \omega_- B_-),$$

which gives

$$A_+ = \frac{\phi_0}{2} - \frac{\psi_0}{2\sqrt{2}},$$
$$A_- = \frac{\phi_0}{2} + \frac{\psi_0}{2\sqrt{2}},$$
$$B_+ = \frac{\dot{\phi}_0}{2\omega_+} - \frac{\dot{\psi}_0}{2\sqrt{2} \omega_+},$$
$$B_- = \frac{\dot{\phi}_0}{2\omega_-} + \frac{\dot{\psi}_0}{2\sqrt{2} \omega_-}.$$

For the initial conditions

$$\phi_0 = 0, \quad \dot{\phi}_0 = 1, \quad \psi_0 = 0, \quad \dot{\psi}_0 = -1$$

at time $t = 0$ the figure gives a plot up to $t = 50 \sqrt{l/g}$ (next page).