Problem 1

Two non-interacting particles inhabit a potential well such that the orbital motion of each particle gives rise to an energy spectrum $E(n) = n\epsilon$, the $n^{th}$ energy level having a degeneracy $g(n) = 2n + 1$. Consider spin-0 particles obeying Bose-Einstein statistics, spin-$\frac{1}{2}$ particles obeying Fermi-Dirac statistics, and spin-$s$ particles obeying Maxwell-Boltzmann statistics. Assuming that the energy is spin-independent,

1.a) find the microcanonical partition function of the system when the total energy has the fixed value $E = N\epsilon$,

1.b) find the canonical partition function of the system when it is in contact with a heat reservoir at fixed temperature $T$, and

1.c) investigate the relationship between the partition functions obtained for the three types of statistics.