

3. Consider the one-loop diagrams with three external photons (and no external fermions) and show that the corresponding amplitude is zero. This is a particular case of Furry’s theorem, for which you may want to solve Problem 58.2).


5. Problem 66.3 of Srednicki’s book.

6. Cancelation of IR divergences in inclusive cross sections.
Consider the scattering process $e^{-}(p) + \gamma(q) \rightarrow e^{-}(p') \ (p^2 = (p')^2 = -m^2, \ q^2 \neq 0)$ discussed in class, and show that the IR divergences arising in the $O(\alpha)$ QED loop corrections cancel in the $O(\alpha^2)$ inclusive cross section via equal and opposite terms present in the one-photon emission contributions. To this extent it can be useful to consider the following series of steps:

6.1 write the $O(\alpha^2)$ cross section as a sum of a tree-level piece ($\sigma_0$) and an $O(\alpha)$ correction ($\sigma_1$) (notice that $\sigma_0$ starts at $O(\alpha)$, and $\sigma_1$ is per se of $O(\alpha^2)$ but represents the $O(\alpha)$ correction to $\sigma_0$);

6.2 show that $\sigma_1$ receives contributions both from one-loop (or virtual) corrections to the $e^{-}(p) + \gamma(q) \rightarrow e^{-}(p')$ process (show/identify them explicitly) and from the tree-level process $e^{-}(p) + \gamma(q) \rightarrow e^{-}(p') + \gamma(k)$ (real photon emission, show diagrams explicitly), where the final state photon might not be identified, hence the meaning of inclusive cross section;

6.3 review and reproduce the calculation of the IR divergences present in the $O(\alpha^2)$ one-loop part of the cross section (I would like you to use dimensional regularization and to show how to calculate the IR-divergent parts, limiting yourself to the pole/singular parts only);

6.4 calculate the cross section for $e^{-}(p) + \gamma(q) \rightarrow e^{-}(p') + \gamma(k)$ in the limit of vanishing photon energy ($k^0 \rightarrow 0$, a.k.a. soft limit) keeping only the IR divergent pieces. It is useful to notice that in this limit both the scattering amplitude and the phase-space integration factorize, such that,

$$\int dPS_{2\rightarrow 2} |A(e^{-}\gamma \rightarrow e^{-}\gamma)|^2 \left. \frac{k^0 \rightarrow 0}{\int dPS_{2\rightarrow 1} \int dPS_{\gamma, soft} |A(e^{-}\gamma \rightarrow e^{-})|^2} \right.$$

(1)
where $S_{\text{soft}}$ is given by,

$$S_{\text{soft}} = \frac{p^2}{(pk)^2} + \frac{(p')^2}{(p'k)^2} - \frac{2(p \cdot p')}{(p \cdot k)(p' \cdot k)},$$  \hspace{1cm} (2)$$

and the photon phase space in $d = 4 - 2\epsilon$ dimensions can be written as,

$$\int dPS_{\gamma} = \int \frac{d^{d-1}k}{(2\pi)^{d-1}2k^0} = \frac{(4\pi)^\epsilon}{8\pi^2} \int_0^{\delta} dk^0(k^0)^{1-2\epsilon} \int_0^{\pi} d\theta \sin^{1-2\epsilon}\theta,$$  \hspace{1cm} (3)$$

where $\delta$ is an arbitrary upper bound on the photon energy.

6.5 You should now have all the information to answer the problem.