1 Graded problems

1. A particle of mass $m$ travels in a hyperbolic orbit past a mass $M$, whose position is assumed to be fixed. The speed at infinity is $v_0$, and the impact parameter is $b$.

   (1.a) Show that the angle through which the particle is deflected is
   \[ \Theta = \pi - 2 \tan^{-1}(\gamma b) \Rightarrow b = \frac{1}{\gamma \cot\left(\frac{\Theta}{2}\right)}, \]
   where $\gamma \equiv \frac{v_0^2}{GM}$.

   (1.b) Let $d\sigma$ be the cross-sectional area (measured when the particle is initially at infinity) that gets deflected into a solid angle of size $d\Omega$ at angle $\Theta$ (this quantity is called differential cross section). Show that
   \[ \frac{d\sigma}{d\Omega} = \frac{1}{4\gamma^2 \sin^4(\Theta/2)}. \]

   (1.c) Consider the case of backward scattering, i.e. $\Theta \approx 180^\circ$. What can you tell in the limiting cases of small $v_0$ ($v_0 \to 0$) and large $v_0$ ($v_0 \to \infty$)? Explain your results.

   (1.d) Consider the case of negligible deflection, i.e. $\Theta \approx 0^\circ$. Does it make sense that $\sigma \approx \infty$ and why? How should the potential behave in order not to generate an infinite cross section?

   (1.e) Show that in case you replace the gravitational force with the electrostatic one (Coulomb interaction between pointlike charges) you get Rutherford-scattering differential cross section:
   \[ \frac{d\sigma}{d\Omega} = \frac{K^2 q_1^2 q_2^2}{16E^2 \sin^4(\Theta/2)}. \]

2. Examine the scattering produced by a repulsive central force $f = kr^{-3}$. Show that the differential cross section is given by
   \[ \sigma(\Theta)d\Theta = \frac{k}{2E} \frac{(1-x)dx}{x^2(2-x)^2 \sin(\pi x)}, \]
   where $x = \Theta/\pi$ and $E$ is the energy.
3. Show that the angle of scattering in the laboratory system, \( \theta \), is related to the energy before scattering (\( E_0 \)) and the energy after scattering (\( E_1 \)) according to the equation

\[
\cos \theta = \frac{m_1 + m_2}{2m_1} \sqrt{\frac{E_1}{E_0}} + \frac{m_1 - m_2}{2m_1} \sqrt{\frac{E_0}{E_1}} + \frac{m_2 Q}{2m_1 \sqrt{E_0 E_1}},
\]

where \( Q \) is the so-called \( Q \)–value of the collision.

2 Non-graded suggested problems