Higgs Boson Physics, Part I

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Outline of Part I

• Understanding the Electroweak Symmetry Breaking as a first step towards a more fundamental theory of particle physics.

• The Higgs mechanism and the breaking of the Electroweak Symmetry in the Standard Model.
  → Toy model: breaking of an abelian gauge symmetry.
  → Quantum effects in spontaneously broken gauge theories.
  → The Standard Model: breaking of the $SU(2)_L \times U(1)_Y$ symmetry.
  → Fermion masses through Yukawa-like couplings to the Higgs field.

• First step: calculate the SM Higgs boson decay branching ratios.
Some References for Part I

- Spontaneous Symmetry Breaking of global and local symmetries:
  - An Introduction to Quantum Field Theory, M.E. Peskin and D.V. Schroeder
  - The Quantum Theory of Fields, V. II, S. Weinberg

- Theory and Phenomenology of the Higgs boson(s):
  - The Higgs Hunter Guide, J. Gunion, H.E. Haber, G. Kane, and S. Dawson
  - Introduction to electroweak symmetry breaking, S. Dawson, hep-ph/9901280
Breaking the Electroweak Symmetry: Why and How?

- The gauge symmetry of the Standard Model (SM) forbids gauge boson mass terms, but:

  \[ M_{W^\pm} = 80.426 \pm 0.034 \text{ GeV} \quad \text{and} \quad M_Z = 91.1875 \pm 0.0021 \text{ GeV} \]

\[ \Downarrow \]

Electroweak Symmetry Breaking (EWSB)

- Broad spectrum of ideas proposed to explain the EWSB:
  - Weakly coupled dynamics embedded into some more fundamental theory at a scale \( \Lambda \) (probably \( \sim \) TeV):
    - \[ \Rightarrow \text{Higgs Mechanism} \] in the SM or its extensions (MSSM, etc.)
    - \[ \rightarrow \] Little Higgs models
  - Strongly coupled dynamics at the TeV scale:
    - \[ \rightarrow \] Technicolor in its multiple realizations.
  - Extra dimensions beyond the 3+1 space-time dimensions
Different but related .....

- Explicit fermion mass terms also violate the gauge symmetry of the SM:
  - introduced through new gauge invariant interactions, as dictated by the mechanism of EWSB
  - intimately related to flavor mixing and the origin of CP-violation: new experimental evidence on this side will give further insight.
The story begins in 1964 . . .

with Englert and Brout; Higgs; Hagen, Guralnik and Kibble
Spontaneous Breaking of a Gauge Symmetry

**Abelian Higgs mechanism:** one vector field $A^\mu(x)$ and one complex scalar field $\phi(x)$:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi$$

where

$$\mathcal{L}_A = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} = -\frac{1}{4} (\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

and $(D^\mu = \partial^\mu + ig A^\mu)$

$$\mathcal{L}_\phi = (D^\mu \phi)^* D_\mu \phi - V(\phi) = (D^\mu \phi)^* D_\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

$\mathcal{L}$ invariant under local phase transformation, or local $U(1)$ symmetry:

$$\phi(x) \rightarrow e^{i\alpha(x)} \phi(x)$$

$$A^\mu(x) \rightarrow A^\mu(x) + \frac{1}{g} \partial^\mu \alpha(x)$$

Mass term for $A^\mu$ breaks the $U(1)$ gauge invariance.
Can we build a gauge invariant massive theory? Yes.

Consider the potential of the scalar field:

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$

where $\lambda > 0$ (to be bounded from below), and observe that:

$$\mu^2 > 0 \quad \rightarrow \quad \text{unique minimum:} \quad \phi^* \phi = 0$$

$$\mu^2 < 0 \quad \rightarrow \quad \text{degeneracy of minima:} \quad \phi^* \phi = -\frac{\mu^2}{2\lambda}$$
• $\mu^2 > 0 \rightarrow$ electrodynamics of a massless photon and a massive scalar field of mass $\mu$ ($g = -e$).

• $\mu^2 < 0 \rightarrow$ when we choose a minimum, the original $U(1)$ symmetry is spontaneously broken or hidden.

$$\phi_0 = \left(-\frac{\mu^2}{2\lambda}\right)^{1/2} = \frac{v}{\sqrt{2}} \rightarrow \phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x))$$

$$\downarrow$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2v^2A^\mu A_\mu + \frac{1}{2}(\partial^\mu \phi_1)^2 + \mu^2 \phi_1^2 + \frac{1}{2}(\partial^\mu \phi_2)^2 + gvA_\mu \partial^\mu \phi_2 + \ldots$$

**Side remark:** The $\phi_2$ field actually generates the correct transverse structure for the mass term of the (now massive) $A^\mu$ field propagator:

$$\langle A^\mu(k)A^\nu(-k) \rangle = \frac{-i}{k^2 - m_A^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) + \cdots$$
More convenient parameterization (unitary gauge):

$$\phi(x) = \frac{e^{i \chi(x)/v}}{\sqrt{2}} (v + H(x)) \xrightarrow{U(1)} \frac{1}{\sqrt{2}} (v + H(x))$$

The $\chi(x)$ degree of freedom (Goldstone boson) is rotated away using gauge invariance, while the original Lagrangian becomes:

$$\mathcal{L} = \mathcal{L}_A + \frac{g^2 v^2}{2} A^\mu A_\mu + \frac{1}{2} \left( \partial^\mu H \partial_\mu H + 2 \mu^2 H^2 \right) + \ldots$$

which describes now the dynamics of a system made of:

- a massive vector field $A^\mu$ with $m_A^2 = g^2 v^2$;
- a real scalar field $H$ of mass $m_H^2 = -2 \mu^2 = 2 \lambda v^2$: the Higgs field.

\[\downarrow\]

Total number of degrees of freedom is balanced
Non-Abelian Higgs mechanism: several vector fields $A^a_\mu(x)$ and several (real) scalar field $\phi_i(x)$:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi, \quad \mathcal{L}_\phi = \frac{1}{2} (D^\mu \phi)^2 - V(\phi), \quad V(\phi) = \mu^2 \phi^2 + \frac{\lambda}{2} \phi^4$$

$(\mu^2 < 0, \lambda > 0)$ invariant under a non-Abelian symmetry group $G$:

$$\phi_i \rightarrow (1 + i\alpha^a t^a)_{ij} \phi_j \quad t^a \rightarrow iT^a = (1 - \alpha^a T^a)_{ij} \phi_j$$

(s.t. $D_\mu = \partial_\mu + g A^a_\mu T^a$). In analogy to the Abelian case:

$$\frac{1}{2} (D_\mu \phi)^2 \rightarrow \ldots + \frac{1}{2} g^2 (T^a \phi)_i (T^b \phi)_i A^a_\mu A^{b\mu} + \ldots$$

$$\phi_{\text{min}} \rightarrow \phi_0 \rightarrow \ldots + \frac{1}{2} g^2 (T^a \phi_0)_i (T^b \phi_0)_i A^a_\mu A^{b\mu} + \ldots = m^2_{ab}$$

\[
\begin{array}{c}
T^a \phi_0 \neq 0 \quad \rightarrow \quad \text{massive vector boson} + \text{(Goldstone boson)} \\
T^a \phi_0 = 0 \quad \rightarrow \quad \text{massless vector boson} + \text{massive scalar field}
\end{array}
\]
Classical $\rightarrow$ Quantum: $V(\phi) \rightarrow V_{eff}(\varphi_{cl})$

The stable vacuum configurations of the theory are now determined by the extrema of the Effective Potential:

$$V_{eff}(\varphi_{cl}) = -\frac{1}{V_T} \Gamma_{eff}[\phi_{cl}] \ , \ \phi_{cl} = \text{constant} = \varphi_{cl}$$

where

$$\Gamma_{eff}[\phi_{cl}] = W[J] - \int d^4y J(y)\phi_{cl}(y) \ , \ \phi_{cl}(x) = \frac{\delta W[J]}{\delta J(x)} = \langle 0|\phi(x)|0 \rangle_J$$

$W[J]$ $\rightarrow$ generating functional of connected correlation functions

$\Gamma_{eff}[\phi_{cl}]$ $\rightarrow$ generating functional of 1PI connected correlation functions

$V_{eff}(\varphi_{cl})$ can be organized as a loop expansion (expansion in $\bar{\hbar}$), s.t.:

$$V_{eff}(\varphi_{cl}) = V(\varphi_{cl}) + \text{loop effects}$$

SSB $\rightarrow$ non trivial vacuum configurations
Gauge fixing: the $R_\xi$ gauges. Consider the abelian case:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D^\mu \phi)^* D_\mu \phi - V(\phi)$$

upon SSB:

$$\phi(x) = \frac{1}{\sqrt{2}} ((v + \phi_1(x)) + i\phi_2(x))$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial^\mu \phi_1 + gA^\mu \phi_2)^2 + \frac{1}{2} (\partial^\mu \phi_2 - gA^\mu (v + \phi_1))^2 - V(\phi)$$

Quantizing using the gauge fixing condition:

$$G = \frac{1}{\sqrt{\xi}} (\partial_\mu A^\mu + \xi g v \phi_2)$$

in the generating functional

$$Z = C \int \mathcal{D}A \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp \left[ \int d^4 x \left( \mathcal{L} - \frac{1}{2} G^2 \right) \right] \det \left( \frac{\delta G}{\delta \alpha} \right)$$

($\alpha \rightarrow$ gauge transformation parameter)
\[
\mathcal{L} - \frac{1}{2}G^2 = -\frac{1}{2}A_\mu \left( -g^{\mu\nu} \partial^2 + \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu - (gv)^2 g^{\mu\nu} \right) A_\nu \\
\frac{1}{2} (\partial_\mu \phi_1)^2 - \frac{1}{2} m_{\phi_1}^2 \phi_1^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 - \frac{\xi}{2} (gv)^2 \phi_2^2 + \cdots \\
+ \\
\mathcal{L}_{\text{ghost}} = \bar{c} \left[ -\partial^2 - \xi (gv)^2 \left( 1 + \frac{\phi_1}{v} \right) \right] c
\]

such that:

\[
\langle A^\mu(k) A^\nu(-k) \rangle = \frac{-i}{k^2 - m_A^2} \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) + \frac{-i\xi}{k^2 - \xi m_A^2} \left( \frac{k^\mu k^\nu}{k^2} \right)
\]

\[
\langle \phi_1(k) \phi_1(-k) \rangle = \frac{-i}{k^2 - m_{\phi_1}^2}
\]

\[
\langle \phi_2(k) \phi_2(-k) \rangle = \langle c(k) \bar{c}(-k) \rangle = \frac{-i}{k^2 - \xi m_A^2}
\]

Goldstone boson $\phi_2$, $\leftrightarrow$ longitudinal gauge bosons
The Higgs sector of the Standard Model: 
\[ SU(2)_L \times U(1)_Y \xrightarrow{\text{SSB}} U(1)_Q \]

Introduce one complex scalar doublet of \( SU(2)_L \) with \( Y = 1/2 \):

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \longleftrightarrow \quad \mathcal{L} = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2
\]

where \( D_\mu \phi = (\partial_\mu - igA^a_\mu \tau^a - ig'Y\phi B_\mu) \), \( (\tau^a = \sigma^a/2, \ a = 1, 2, 3) \).

The SM symmetry is spontaneously broken when \( \langle \phi \rangle \) is chosen to be (e.g.):

\[
\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with} \quad v = \left( \frac{-\mu^2}{\lambda} \right)^{1/2} \quad (\mu^2 < 0, \ \lambda > 0)
\]

The gauge boson mass terms arise from:

\[
(D^\mu \phi)^\dagger D_\mu \phi \quad \longrightarrow \quad \cdots + \frac{1}{8} (0 \ v) \left( gA^a_\mu \sigma^a + g' B_\mu \right) \left( gA^{b\mu} \sigma^b + g' B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} + \cdots
\]

\[
\longrightarrow \quad \cdots + \frac{1}{2} \frac{v^2}{4} \left[ g^2 (A^1_\mu)^2 + g^2 (A^2_\mu)^2 + (-gA^3_\mu + g' B_\mu)^2 \right] + \cdots
\]
And correspond to the weak gauge bosons:

\[
W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \pm A^2_\mu) \quad \rightarrow \quad M_W = \frac{g^v}{2}
\]

\[
Z^0_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(gA^3_\mu - g'B_\mu) \quad \rightarrow \quad M_Z = \sqrt{g^2 + g'^2} \frac{v}{2}
\]

while the linear combination orthogonal to \(Z^0_\mu\) remains massless and corresponds to the photon field:

\[
A_\mu \frac{1}{\sqrt{g^2 + g'^2}}(g'A^3_\mu + gB_\mu) \quad \rightarrow \quad M_A = 0
\]

Notice: using the definition of the weak mixing angle, \(\theta_w\):

\[
\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}} \quad , \quad \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}
\]

the \(W\) and \(Z\) masses are related by:

\[
M_W = M_Z \cos \theta_w
\]
The scalar sector becomes more transparent in the unitary gauge:

\[
\phi(x) = e^{i v \vec{\chi}(x) \cdot \vec{\tau}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \xrightarrow{SU(2)} \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}
\]

after which the Lagrangian becomes

\[
\mathcal{L} = \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \sqrt{\frac{\lambda}{2}} M_H H^3 - \frac{1}{4} \lambda H^4
\]

Three degrees of freedom, the \(\chi^a(x)\) Goldstone bosons, have been reabsorbed into the longitudinal components of the \(W^\pm\) and \(Z^0\) weak gauge bosons. One real scalar field remains:

the Higgs boson, \(H\), with mass \[M_H^2 = -2 \mu^2 = 2 \lambda v^2\]

and self-couplings:

\[-3i \frac{M_H^2}{v^2}\]
From \((D\mu\phi)\dagger D_\mu\phi \longrightarrow\) Higgs-Gauge boson couplings:

\[
V^\mu \rightarrow H = 2i \frac{M^2_H}{v} g^{\mu\nu} \\
V^\nu \rightarrow HH = 2i \frac{M^2_H}{v^2} g^{\mu\nu}
\]

Notice: The entire Higgs sector depends on only two parameters, e.g. \(M_H\) and \(v\)

\(v\) measured in \(\mu\)-decay: \(v = (\sqrt{2}G_F)^{-1/2} = 246\) GeV → **SM Higgs Physics depends on \(M_H\)**
Also: remember Higgs-gauge boson loop-induced couplings:

They will be discussed in the context of Higgs boson decays.
Finally: Higgs boson couplings to quarks and leptons

The gauge symmetry of the SM also forbids fermion mass terms $(m_{Q_i} Q_i^L u_R^i, \ldots)$, but all fermions are massive.

\[ \downarrow \]

Fermion masses are generated via gauge invariant Yukawa couplings:

\[ \mathcal{L}_{Yukawa} = -\Gamma^{ij}_u \bar{Q}^i_L \phi^{c} u^j_R - \Gamma^{ij}_d \bar{Q}^i_L \phi^{d} d^j_R - \Gamma^{ij}_e \bar{L}^i_L \phi^{l} l^j_R + h.c. \]

such that, upon spontaneous symmetry breaking:

\[ \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \quad \rightarrow \quad m_f = \Gamma f \frac{v}{\sqrt{2}} \]

and

\[ f \quad \overrightarrow{H} \quad \overleftarrow{f} \quad = -i \frac{m_f}{v} = -i y_t \]
SM Higgs boson decay branching ratios

We can now calculate branching ratios and total width of the SM Higgs boson:

![Graph showing branching ratios and total width of the SM Higgs boson](image)

Observe difference between light and heavy Higgs

These curves include: tree level + QCD and EW loop corrections.
Tree level decays: $H \to f \bar{f}$ and $H \to VV$

At lowest order:

$$\Gamma(H \to f \bar{f}) = \frac{G_F M_H}{4\sqrt{2}\pi} N_c f m_f^2 \beta_f^3$$

$$\Gamma(H \to VV) = \frac{G_F M_H^3}{16\sqrt{2}\pi} \delta_V \left( 1 - \tau_V + \frac{3}{4} \tau_V^2 \right) \beta_V$$

($\beta_i = \sqrt{1 - \tau_i}$, $\tau_i = 4m_i^2/M_H^2$, $\delta_{W,Z} = 2, 1$, $(N_c)_{l,q} = 1, 3$)

**Ex.1:** Higher order corrections to $H \to q\bar{q}$

QCD corrections dominant:

$$\Gamma(H \to q\bar{q})_{QCD} = \frac{3G_F M_H}{4\sqrt{2}\pi} \bar{m}_q^2(M_H) \beta_q^3 [\Delta_{QCD} + \Delta_t]$$

$$\Delta_{QCD} = 1 + 5.67 \frac{\alpha_s(M_H)}{\pi} + (35.94 - 1.36 N_F) \left( \frac{\alpha_s(M_H)}{\pi} \right)^2$$

$$\Delta_t = \left( \frac{\alpha_s(M_H)}{\pi} \right)^2 \left[ 1.57 - \frac{2}{3} \ln \frac{M_H^2}{m_t^2} + \frac{1}{9} \ln^2 \frac{\bar{m}_q^2(M_H)}{M_H^2} \right]$$
Consist of both virtual and real corrections:
• Large Logs absorbed into $\overline{MS}$ quark mass

**Leading Order:**
$$m_Q(\mu) = m_Q(m_Q) \left( \frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right)^{\frac{2b_0}{\gamma_0}}$$

**Higher order:**
$$m_Q(\mu) = m_Q(m_Q) \frac{f(\alpha_s(\mu)/\pi)}{f(\alpha_s(m_Q)/\pi)}$$

where (from renormalization group equation)

$$f(x) = \left( \frac{25}{6} \right)^{\frac{12}{25}} x^{\frac{12}{25}} \left[ 1 + 1.014x + \ldots \right] \quad \text{for} \quad m_c < \mu < m_b$$

$$f(x) = \left( \frac{23}{6} \right)^{\frac{12}{23}} x^{\frac{12}{23}} \left[ 1 + 1.175x + \ldots \right] \quad \text{for} \quad m_b < \mu < m_t$$

$$f(x) = \left( \frac{7}{2} \right)^{\frac{4}{7}} x^{\frac{4}{7}} \left[ 1 + 1.398x + \ldots \right] \quad \text{for} \quad \mu > m_t$$

• Large corrections, when $M_H \gg m_Q$

$$m_b(m_b) \approx 4.2 \text{ GeV} \quad \rightarrow \quad \tilde{m}_b(M_h \approx 100 \text{ GeV}) \approx 3 \text{ GeV}$$

Branching ratio smaller by almost a factor 2.

• Main uncertainties: $\alpha_s(M_Z)$, pole masses: $m_c(m_c)$, $m_b(m_b)$. 

Ex. 2: Higher order corrections \( \Gamma(H \rightarrow gg) \)

Start from tree level:

\[
\Gamma(H \rightarrow gg) = \frac{G_F \alpha_s^2 M_H^3}{36 \sqrt{2} \pi^3} \left| \sum_q A_q^H (\tau_q) \right|
\]

where \( \tau_q = \frac{4 m_q^2}{M_H^2} \) and

\[
A_q^H (\tau) = \frac{3}{2} \tau \left[ 1 + (1 - \tau) f(\tau) \right]
\]

\[
f(\tau) = \begin{cases} 
\arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\
-\frac{1}{4} \left[ \ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right]^2 & \tau < 1 
\end{cases}
\]

Main contribution from top quark \( \rightarrow \) optimal situation to use Low Energy Theorems to add higher order corrections.
QCD corrections dominant:

Difficult task since decay is already a loop effect.

However, full massive calculation of $\Gamma(H \to gg(q), q\bar{q}g)$ agrees with $m_t \gg M_H$ result at 10%

$$\Gamma(H \to gg(q), q\bar{q}g) = \Gamma_{LO}(\alpha_s^{(N_L)}(M_H)) \left[ 1 + E^{(N_L)} \frac{\alpha_s^{(N_L)}}{\pi} \right]$$

$$E^{(N_L)} \xrightarrow{M_H^2 \ll 4m_q^2} \frac{95}{4} - \frac{7}{6} N_L$$

Dominant soft/collinear radiation do not resolve the Higgs boson coupling to gluons $\quad \longrightarrow \quad$ QCD corrections are just a (big) rescaling factor
NLO QCD corrections almost 60 – 70% of LO result in the low mass region:

solid line $\rightarrow$ full massive NLO calculation

dashed line $\rightarrow$ heavy top limit ($M_H^2 \ll 4m_t^2$)

NNLO corrections calculated in the heavy top limit: add 20%

$\rightarrow$ perturbative stabilization.
Low-energy theorems, in a nutshell.

- **Observing that:**

  In the $p_H \to 0$ limit: the interactions of a Higgs boson with the SM particles arise by substituting

  $$
  M_i \longrightarrow M_i \left(1 + \frac{H}{v}\right) \quad (i = f, W, Z)
  $$

  In practice: Higgs taken on shell ($p_H^2 = M_H^2$), and limit $p_H \to 0$ is limit of small Higgs masses (e.g.: $M_H^2 \ll 4m_t^2$).

- **Then**

  $$
  \lim_{p_H \to 0} A(X \to Y + H) = \frac{1}{v} \sum_i M_i \frac{\partial}{\partial M_i} A(X \to Y)
  $$

  very convenient!

- **Equivalent to an Effective Theory** described by:

  $$
  \mathcal{L}_{e.f.f} = \frac{\alpha_s}{12\pi} G^{a \mu \nu} G_{a \mu \nu} \frac{H}{v} (1 + O(\alpha_s))
  $$

  including higher order QCD corrections.
For completeness:

\[
\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128 \sqrt{2}\pi^3} \left| \sum_f N_c f e_f^2 A_f^H (\tau_f) + A_W^H (\tau_W) \right|^2
\]

where \((f(\tau) \text{ as in } H \to gg)\):

\[
A_f^H = 2\tau \left[ 1 + (1 - \tau) f(\tau) \right]
\]

\[
A_W^H (\tau) = - \left[ 2 + 3\tau + 3\tau(2 - \tau) f(\tau) \right]
\]

\[
\Gamma(H \to Z\gamma) = \frac{G_F^2 M_W^2 \alpha M_H^3}{64\pi^4} \left( 1 - \frac{M_Z^2}{M_H^2} \right)^3 \left| \sum_f A_f^H (\tau_f, \lambda_f) + A_W^H (\tau_W, \lambda_W) \right|^2
\]

where the form factors \(A_f^H (\tau, \lambda)\) and \(A_W^H (\tau, \lambda)\) can be found in the literature (see, e.g., M. Spira, hep-ph/9705337).

For both decays, both QCD and EW corrections are very small \((\approx 1 - 3\%)\).
Present theoretical accuracy on SM Higgs branching ratios

Example: $M_H = 120$ GeV

<table>
<thead>
<tr>
<th>Decay mode</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b\bar{b}$</td>
<td>1.4%</td>
</tr>
<tr>
<td>$WW^*$</td>
<td>2.3%</td>
</tr>
<tr>
<td>$\tau^+\tau^-$</td>
<td>2.3%</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>23%</td>
</tr>
<tr>
<td>$gg$</td>
<td>5.7%</td>
</tr>
<tr>
<td>$\gamma\gamma$</td>
<td>2.3%</td>
</tr>
</tbody>
</table>

Mainly due to: pole masses $m_c$ and $m_b$, and $\alpha_s(\mu)$. 