Higgs Boson Physics, Part IV

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Outline of Part IV

• The current status of theoretical predictions in Higgs physics: How does theory compare to experiments?

• Highlights from the theoretical activity of the last few years:
  → $gg \rightarrow H$: gluon-gluon fusion calculated at NNLO in QCD, consolidating the reliability of a very difficult theoretical prediction.
  → $q\bar{q}, gg \rightarrow t\bar{t}H$ calculated at NLO in QCD, stabilizing an important theoretical prediction for Higgs discovery in the (most interesting) low mass region.
  → $q\bar{q}, gg \rightarrow b\bar{b}H, bg \rightarrow bH, b\bar{b} \rightarrow H$: now available including NLO (NNLO) QCD corrections, which clarify a very controversial theoretical scenario.

• Final Overview
Understanding the current status of theoretical predictions

or

How theory can compare to experiments
The basic picture of a $p\bar{p}, pp \rightarrow X$ high energy process . . .

where the short and long distance part of the QCD interactions can be factorized and the cross section for $pp, p\bar{p} \rightarrow X$ can be calculated as:

$$\sigma(pp, p\bar{p} \rightarrow X) = \sum_{ij} \int dx_1 dx_2 f_i^p(x_1) f_j^{p,\bar{p}}(x_2) \hat{\sigma}(ij \rightarrow X)$$

$\rightarrow ij \rightarrow$ quarks or gluons (partons)
$\rightarrow f_i^p(x), f_i^{p,\bar{p}}(x)$: Parton Distributions Functions: probability densities (probability of finding parton $i$ in $p$ or $\bar{p}$ with a fraction $x$ of the original hadron momentum)
$\rightarrow \hat{\sigma}(ij \rightarrow X)$: partonic cross section
... is complicated by the presence of interactions

→ Focus on strong interactions, dominant at hadron colliders

→ In the $ij \rightarrow X$ process, initial and final state partons radiate and absorb gluons/quarks:

How to calculate the physical cross section?

→ Due to the very same interactions: the strong coupling constant ($\alpha_s = g_s^2/4\pi$) becomes a function of the energy scale ($Q^2$), such that

$$\alpha_s(Q^2) \rightarrow 0 \quad \text{for large scales } Q^2 : \text{ running coupling}$$

\[ \downarrow \]

we can calculate $\hat{\sigma}(ij \rightarrow X)$ perturbatively

$$\hat{\sigma}(ij \rightarrow X) = \alpha_s^k \sum_{m=0}^{n} \hat{\sigma}_{ij}^{(m)} \alpha_s^m$$

n=0 : Leading Order (LO), or tree level or Born level

n=1 : Next to Leading Order (NLO), include $O(\alpha_s)$ corrections

........
Perturbative approach and scale dependence

→ At each order in $\alpha_s$ the expression of $\hat{\sigma}(ij \rightarrow X)$ contains infinities that are canceled by a subtraction procedure: renormalization.

→ A remnant of the subtraction point is left at each perturbative order as a renormalization scale dependence ($\mu_R$)

$$
\hat{\sigma}(ij \rightarrow X) = \alpha_k^s(\mu_R) \sum_{m=0}^{n} \hat{\sigma}_{ij}^{(m)}(\mu_R, Q^2) \alpha_s^m(\mu_R)
$$

→ A similar approach introduces a subtraction point dependence in the initial state parton densities: factorization scale dependence ($\mu_F$)

Setting $\mu_R = \mu_F = \mu$

$$
\sigma(pp, p\bar{p} \rightarrow X) = \sum_{ij} \int dx_1 dx_2 f_i^p(x_1, \mu) f_j^{p,\bar{p}}(x_2, \mu) \sum_{m=0}^{n} \hat{\sigma}_{ij}^{(m)}(\mu, Q^2) \alpha_s^{m+k}(\mu)
$$

The theoretical error is systematically organized as an expansion in $\alpha_s$
**Ex.**: General structure of a NLO calculation

NLO total cross section:

\[ \sigma_{pp,pp}^{NLO} = \sum_{i,j} \int dx_1 dx_2 F_i^p(x_1, \mu_F) F_j^{\bar{p},p}(x_2, \mu_F) \delta_{ij}^{NLO}(x_1, x_2, \mu_R, \mu_F) \]

where

\[ \hat{\sigma}_{ij}^{NLO} = \hat{\sigma}_{ij}^{LO} + \frac{\alpha_s}{4\pi} \delta_{ij}^{NLO} \]

NLO corrections made of:

\[ \delta\hat{\sigma}_{ij}^{NLO} = \hat{\sigma}_{ij}^{virt} + \hat{\sigma}_{ij}^{real} \]

- \( \hat{\sigma}_{ij}^{virt} \): one loop virtual corrections.
- \( \hat{\sigma}_{ij}^{real} \): one gluon/quark real emission.
- use \( \alpha_s^{NLO}(\mu) \) and match with NLO PDF’s.

\[ \rightarrow \text{renormalize UV divergences } (d=4-2\epsilon_{UV}) \]
\[ \rightarrow \text{cancel IR divergences in } \hat{\sigma}_{virt} + \hat{\sigma}_{real} \quad (d=4-2\epsilon_{IR}) \]
\[ \rightarrow \text{check } \mu\text{-dependence of } \sigma^{NLO}_{pp,pp}(\mu_R,\mu_F) \]
Some important facts . . .

- In hadronic production modes QCD effects can be very large.

- Tree level or Leading Order (LO) cross sections normally have very large uncertainties due to:
  - renormalization/factorization scale dependence
  - uncertainties from PDF’s

- Differential cross sections very sensitive to higher order QCD corrections.

- Main modes have very large QCD background. Realistic studies, including higher order QCD corrections, are vital to a correct estimate of the background.

Crucial to know Higher Order QCD corrections
Some References for Part IV

State-of-the-art of QCD predictions for Higgs production at Hadron Colliders.

<table>
<thead>
<tr>
<th>process</th>
<th>$\sigma_{NLO,NNLO}$ by</th>
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                  | C.J.Glosser et al., JHEP (2002); V.Ravindran et al., NPB 634 (2002)  
                  | D. de Florian et al., PRL 82 (1999)  
                  | C.Anastasiou, K.Melnikov, NPB 646 (2002) (NNLO)  
                  | V.Ravindran et al., NPB 665 (2003) (NNLO)  
| $q\bar{q} \to (W, Z)H$ | T.Han, S.Willenbrock, PLB 273 (1991)  
| $q\bar{q} \to q\bar{q}H$ | T.Han, G.Valencia, S.Willenbrock, PRL 69 (1992)  
| $q\bar{q}, gg \to t\bar{t}H$ | W.Beenakker et al., PRL 87 (2001), NPB 653 (2003)  
| $q\bar{q}, gg \to b\bar{b}H$ | S.Dittmaier, M.Krämer, M.Spira, hep-ph/0309204  
                                | S.Dawson et al., PRD 69 (2004) |
| $gb(b) \to b(\bar{b})H$ | J.Cambell et al., PRD 67 (2003) |
| $b\bar{b}H$ | D.A.Dicus et al. PRD 59 (1999); C.Balasz et al., PRD 60 (1999).  
Highlights of recent theoretical activity

- $H \rightarrow gg$ at NNLO+NNLL
- $p\bar{p}, pp \rightarrow t\bar{t}H$ at NLO
- $p\bar{p}, pp \rightarrow b\bar{b}H$ at NLO

Apologies for what will be omitted …
Preliminaries: Higher order corrections $\Gamma(H \rightarrow gg)$

Start from tree level:

\[
\Gamma(H \rightarrow gg) = \frac{G_F \alpha_s^2 M_H^3}{36 \sqrt{2} \pi^3} \left| \sum_q A_q^H(\tau_q) \right|^2
\]

where $\tau_q = 4m_q^2/M_H^2$ and

\[
A_q^H(\tau) = \frac{3}{2} \tau \left[ 1 + (1 - \tau)f(\tau) \right]
\]

\[
f(\tau) = \begin{cases} 
\arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\
-\frac{1}{4} \left[ \ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right]^2 & \tau < 1 
\end{cases}
\]

Main contribution from top quark $\rightarrow$ optimal situation to use Low Energy Theorems to add higher order corrections.
QCD corrections dominant:

$$\Gamma(H \rightarrow gg(q), q\bar{q}g) = \Gamma_{LO}(\alpha_s^{(NL)}(M_H)) \left[ 1 + E^{(NL)} \frac{\alpha_s^{(NL)}}{\pi} \right]$$

$$E^{(NL)} \xrightarrow{M_H \ll 4m_q^2} \frac{95}{4} - \frac{7}{6} N_L$$

Difficult task since decay is already a loop effect.

However, full massive calculation of $\Gamma(H \rightarrow gg(q), q\bar{q}g)$ agrees with $m_t \gg M_H$ result at 10%

Dominant soft/collinear radiation do not resolve the Higgs boson coupling to gluons $\xrightarrow{\text{QCD corrections are just a (big) rescaling factor}}$
Low-energy theorems, in a nutshell.

- Observing that:
  In the $p_H \to 0$ limit: the interactions of a Higgs boson with the SM particles arise by substituting

$$M_i \longrightarrow M_i \left(1 + \frac{H}{v}\right) \quad (i = f, W, Z)$$

In practice: Higgs taken on shell ($p_H^2 = M_H^2$), and limit $p_H \to 0$ is limit of small Higgs masses (e.g.: $M_H^2 \ll 4m_t^2$).

- Then

$$\lim_{p_H \to 0} \mathcal{A}(X \to Y + H) = \frac{1}{v} \sum_i M_i \frac{\partial}{\partial M_i} \mathcal{A}(X \to Y)$$

very convenient!

- Equivalent to an **Effective Theory** described by:

$$\mathcal{L}_{e,f,f} = \frac{\alpha_s}{12\pi} G^{\alpha\mu\nu} G_{\mu\nu}^\alpha \frac{H}{v} (1 + O(\alpha_s))$$

including higher order QCD corrections.
$gg \rightarrow H$, the leading production mode

\[ \sigma_{LO} = \frac{G_F\alpha_s(\mu)^2}{288\sqrt{2}\pi} \left| \sum_q A_q^H(\tau_q) \right|^2 \]

\[ \tau_q = \frac{4m_q^2}{M_H^2} \]

\[ A_q^H(\tau_q) \rightarrow 1 \text{ for } \tau_q \rightarrow \infty \]

NLO QCD corrections calculated exactly and in the $m_t \rightarrow \infty$ limit.

where the $K$-factor is defined as:

\[ K(m_t = \infty) = \lim_{m_t \rightarrow \infty} K = \lim_{m_t \rightarrow \infty} \frac{\sigma_{NLO}}{\sigma_{LO}} \]

perfect agreement even for $M_H \gg m_t$

\[ \Downarrow \]

soft radiation dominates
As for $H \rightarrow gg$, this suggest to use at NNLO:

$\rightarrow$ low energy theorems ($m_t \rightarrow \infty$)

\[ \downarrow \]

\[ \mathcal{L}_{\text{eff}} = \frac{H}{4v} C(\alpha_s) G^{a\mu\nu} G^a_{\mu\nu} \]

where, including NLO and NNLO QCD corrections:

\[ C(\alpha_s) = \frac{1}{3} \frac{\alpha_s}{\pi} \left[ 1 + c_1 \frac{\alpha_s}{\pi} + c_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \cdots \right] \]

This reduces by one the order of loops.

Ex.: at NNLO

\[ \downarrow \]

calculates 2-loop diagrams
soft limit \((x = \frac{M_H^2}{\hat{s}} \to 1)\), where

\[
\hat{\sigma}_{ij} = \sum_{n \geq 0} \left( \frac{\alpha_s}{\pi} \right)^n \hat{\sigma}_{ij}^{(n)}
\]

is calculated as an expansion around the soft limit:

\[
\hat{\sigma}_{ij}^{(n)} = a^{(n)} \delta(1 - x) + \sum_{k=0}^{2n-1} b_k^{(n)} \left[ \frac{\ln^k(1-x)}{1-x} \right] + \sum_{l=0}^{\infty} \sum_{k=0}^{2n-1} c_{lk}^{(n)} (1-x)^l \ln^k(1-x)
\]

\[\text{purely soft terms}\]

\[\text{collinear+hard terms}\]

NNLO \(\to a^{(2)}, b_k^{(2)}, c_{lk}^{(2)}\)

(for \(l \geq 0\) and \(k = 0, \ldots, 3\)).

\[\downarrow\]

Remarkable convergence

Confirmed by exact calculation.
Dramatically improved perturbative behavior

\[ \sigma(pp \rightarrow H+X) \text{ [pb]} \]

\[ M_H \text{ [GeV]} \]

\[ \sqrt{s} = 14 \text{ TeV} \]

convergence in going:

\[ \text{LO} \rightarrow \text{NLO} \rightarrow \text{NNLO} \]

Confirmed by the full \[ \mu_R/\mu_F \] dependence:
A comment on K-factors...

\[ K(p\bar{p} \rightarrow H+X) \]

\[ \sqrt{s} = 2 \text{ TeV} \]

\[ \sqrt{s} = 14 \text{ TeV} \]
Further improvement: resumming soft logs

\[ \sigma^{(\text{res})}(s, M_H^2) = \sigma^{(\text{SV})}(s, M_H^2) + \sigma^{(\text{match})}(s, M_H^2) \]

with **NNLO+NNLL**: Theoretical uncertainty reduced to:

\[ \rightarrow \simeq 10\% \text{ perturbative uncertainty, including the } m_t \rightarrow \infty \text{ approximation.} \]

\[ \rightarrow \simeq 10\% \text{ from (now existing, but still to be tested) NNLO PDF’s.} \]
Resummation crucial in transverse momentum distributions

$q_T \rightarrow$ Higgs boson
transverse momentum

large $q_T \quad q_T > M_H$
perturbative expansion
in $\alpha_s(\mu)$

small $q_T \quad q_T \ll M_H$
need to resum large
$\ln(M_H^2/q_T^2)$
Higgs boson production with Heavy Quarks pairs:
\[ p\bar{p}, pp \rightarrow t\bar{t}H, b\bar{b}H \]

Last NLO calculation to be completed, mainly due to:

\[ \rightarrow \] many external particles (2 → 3 process)

\[ \rightarrow \] many massive particles

Still, NLO calculation badly needed: very strong \( \mu_R/\mu_F \) dependence at LO.

\[ \sqrt{s} = 2 \text{ TeV} \]
\[ pp \rightarrow tth_{SM} \]

\[ \sqrt{s} = 14 \text{ TeV} \]
\[ pp \rightarrow tth_{SM} \]
$p\bar{p}, pp \rightarrow t\bar{t}H$: tree level

$q\bar{q} \rightarrow t\bar{t}H$
leading contribution at the Tevatron

$gg \rightarrow t\bar{t}H$
leading contribution at the LHC
$p\bar{p}, pp \rightarrow Q\bar{Q}H: \mathcal{O}(\alpha_s)$ virtual corrections

Main challenge: Pentagon scalar and tensor integrals with several external and internal massive particles

- **Scalar pentagon integrals**: reduced to linear combination of five box scalar integrals (Z.Bern, L.J.Dixon, D.A.Kosower; A.Denner)
- **Tensor pentagon integrals**: numerical instabilities (due to Gram determinant spurious singularities) treated both analytically and numerically.
gives origin to scalar integrals of the form:

\[ E_0 = \frac{1}{16\pi^2 (4\pi\mu^2)^\epsilon \Gamma(3 + \epsilon)} \int d^5 a_i \frac{\delta(1 - \sum_{i=1}^5 a_i)}{[D(a_i)]^{3+\epsilon}} \]

where \( D(a_i) = \sum_{i,j=1}^5 S_{ij} a_i a_j \)

\[
S_{ij} = \frac{1}{2} (M_i^2 + M_j^2 - p_{ij}) \\
p_{ij} = k_i + k_{i+1} + \cdots + k_{j-1}
\]

\[
I_n = \frac{(-1)^n}{2} \left[ \sum_{i=1}^n c_i I_{n-1}^{(i)} + (n - 5 + 2\epsilon)c_0 I_{n-2}^{6-2\epsilon} \right]
\]

\[
E_0 = -\frac{1}{2} \left[ \sum_{i=1}^5 c_i D0^{(i)} + 2\epsilon c_0 E0^{6-2\epsilon} \right]
\]
Pentagon tensor integrals, e.g.:

\[
\begin{align*}
E_1^\mu &= E_1^{(1)} p_1^\mu + E_1^{(2)} p_2^\mu + E_1^{(3)} p_3^\mu + E_1^{(4)} p_4^\mu \\
E_2^{\mu\nu} &= E_2^{(11)} p_1^\mu p_1^\nu + \ldots + E_2^{(12)} (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) + \ldots \\
E_3^{\mu\nu\rho} &= \ldots
\end{align*}
\]

\(E_1^{(i)}, 2, 3\) \(\rightarrow\) determined through Passarino-Veltman reduction: proportional to inverse powers of the full Gram determinant (GD)

\[GD(p_1 + p_2 \rightarrow p_3 + p_4 + p_5) = \det(p_i \cdot p_j) \simeq f(E_3, E_4, \sin \theta_3, \sin \theta_4, \sin \phi_4)\]

\(GD \rightarrow 0\) when two momenta become degenerate: spurious divergences.

We use two methods:

\(\rightarrow\) Kinematic cuts to avoid numerical instabilities and extrapolation to the unsafe region using several algorithms.

\(\rightarrow\) Eliminate all pentagon tensor integrals at the level of the amplitude square.
\[ pp, pp \to Q\bar{Q}H: \mathcal{O}(\alpha_s) \text{ real corrections} \]

Real gluon emission: IR singularities for 2 \to 4 process

Phase Space Slicing \(\to\) isolate the region of the \(Q\bar{Q}h + g(q, \bar{q})\) phase space where \(s_{ik} \to 0\)

\[ s_{ik} = 2p_i \cdot k = 2E_i k^0 (1 - \beta_i \cos \theta_{ik}) \]

\[ \to \text{two cut-off method:} \begin{cases} \\
\delta_s \ (k^0 < \delta_s \sqrt{s}/2) \\
\delta_c \ (1 - \cos \theta_{ik}) < \delta_c \end{cases} \]

\[ \to \text{one cut-off method:} \ s_{min} \ (s_{ik} < s_{min}) \]

- 2CM : L.Bergmann, J.Owens, B.Harris (review), . . .
- 1CM : W.Giele, N.Glover, D.A.Kosower, S.Keller, E.Laenen
In the Two Cutoff Method \((\delta_s, \delta_c)\):

\[
\hat{\sigma}_{\text{real}}(ij \rightarrow Q\bar{Q}H + g) = \hat{\sigma}_{\text{soft}} + \hat{\sigma}_{\text{hard/coll}} + \hat{\sigma}_{\text{hard/non-coll}}
\]

where

- \(\hat{\sigma}_{\text{soft}} \rightarrow E_g < \frac{\sqrt{s}}{2} \delta_s\)
- \(\hat{\sigma}_{\text{hard/coll}} \rightarrow E_g > \frac{\sqrt{s}}{2} \delta_s\) and \((1 - \cos \theta_{ig}) < \delta_c\)

are computed analytically to extract IR singularities

\[
\hat{\sigma}_{\text{soft}} \propto \int d(P S_3) \left( \int d(P S_g)_{\text{soft}} |A_{\text{soft}}(ij \rightarrow Q\bar{Q}H + g)|^2 \right)
\]

\[
\hat{\sigma}_{\text{hard/coll}} \propto \int d(P S_3) \left( \int d(P S_g)_{\text{coll}} |A_{\text{coll}}(ij \rightarrow Q\bar{Q}H + g)|^2 \right)
\]

- \(\hat{\sigma}_{\text{hard/non-coll}} \rightarrow E_g > \frac{\sqrt{s}}{2} \delta_s\) and \((1 - \cos \theta_{ig}) > \delta_c\)

is computed numerically, since IR finite.
In the One Cutoff Method ($s_{min}$):

$$\hat{\sigma}_{real}(ij \rightarrow Q\bar{Q}H + g) = \hat{\sigma}_{ir} + \hat{\sigma}_{hard}$$

where

- $\hat{\sigma}_{ir} \rightarrow s_{ig} < s_{min}$

is computed analytically to extract IR singularities

$\rightarrow$ cross all colored particles to final state
$\rightarrow$ work with color ordered amplitudes: easier matching between soft and collinear region
$\rightarrow$ crossing functions: to account for difference between i.s. and f.s. collinear singularities

- $\hat{\sigma}_{hard} \rightarrow s_{ig} > s_{min}$

is computed numerically, since IR finite.
Consider e.g. \( gg \rightarrow t\bar{t}H + g \):

\[
\hat{\sigma}_{\text{real}}^{H\rightarrow ggt\bar{t}+g} = \int d(PS_5) \sum |A_{\text{real}}^{H\rightarrow ggt\bar{t}+g}|^2 ,
\]

where

\[
A_{\text{real}}^{H\rightarrow ggt\bar{t}+g} = \sum_{i,j,k=A,B,C \atop i \neq j \neq k} A_{ijk} T^i T^j T^k .
\]

The amplitude square is made of three terms:

\[
\sum |A_{\text{real}}^{H\rightarrow ggt\bar{t}+g}|^2 = \frac{(N^2 - 1)}{2} \left[ \frac{N^2}{4} \sum_{i,j,k=A,B,C \atop i \neq j \neq k} |A_{ijk}|^2 
\right.
\]

\[
- \frac{1}{4} \sum_{i,j,k=A,B,C \atop i \neq j \neq k} |A_{ijk} + A_{ikj} + A_{kij}|^2 + \frac{1}{4} \left( 1 + \frac{1}{N^2} \right) \sum_{i,j,k=A,B,C \atop i \neq j \neq k} |A_{ijk}|^2
\]

with very definite soft/collinear factorization properties.
Soft singularities \(\rightarrow\) straightforward

How to disentangle soft vs collinear region of PS with one cutoff?

Collinear limit for \(ig \rightarrow i': s_{ig} \rightarrow 0\) \((i=g_1, g_2)\)

\[
\begin{align*}
    p_i &= zp'_i \\
    p_g &= (1 - z)p'_i
\end{align*}
\]

Each \(A_{ijk}\) (or linear combination of) proportional to \((s_{ai}s_{ig}s_{gb})^{-1}\)

\[
\begin{align*}
    \text{collinear region} & \quad \begin{cases} 
    s_{ig} < s_{\min} \\
    s_{ai} > s_{\min} \implies zs_{ai'} > s_{\min} \implies z > z_1 = \frac{s_{\min}}{s_{ai'}} \\
    s_{gb} > s_{\min} \implies (1 - z)s_{i'b} > s_{\min} \implies z < 1 - z_2 = 1 - \frac{s_{\min}}{s_{i'b}}
    \end{cases} \\
    z_1, 1 - z_2 \implies \text{integration boundaries}
\end{align*}
\]

How to match with \(\hat{\sigma}_{hard}\)? Match each term in \(|A_{real}|^2\) separately.
Cross section independent of the unphysical cutoff

(LHC, $M_H = 120$ GeV, $\mu = m_t + M_H/2$)
LHC, $pp \rightarrow t\bar{t}H$: NLO cross section

$\sqrt{s}=2$ TeV
$M_h=120$ GeV
$\mu_0=m_t+M_h/2$
CTEQ4 PDF’s

$\sqrt{s}=14$ TeV
$M_h=120$ GeV
$\mu_0=m_t+M_h/2$
CTEQ5 PDF’s

$K = \frac{\sigma_{NLO}}{\sigma_{LO}} < 1$

for most values of $\mu$ and $M_H$

$K = \frac{\sigma_{NLO}}{\sigma_{LO}} > 1$

for most values of $\mu$ and $M_H$

Theoretical uncertainty reduced to about 15%
$pp, pp \rightarrow b\bar{b}H$: exclusive vs inclusive cross section

- **b-quarks identification** requires tagging ($p_T^b$ and $\eta^b$ cuts): exclusive (1 b-tag, 2 b-tags) vs inclusive (0 b-tags) cross section.

- **Large collinear** $\ln(\mu_H^2/m_b^2)$ arise in $gg \rightarrow b\bar{b}h$ when final state b-quarks are collinear to initial state gluons

  \[
  g \xrightarrow{\text{collinear logs}} b\bar{b} \xrightarrow{\text{ regulated by } m_b}
  \]

- **Fully exclusive** cross section (2 b-tags):
  \[
  \rightarrow \text{ needs } gg \rightarrow b\bar{b}h: \text{ two b's in final state}
  \]
  \[
  \rightarrow \text{ has no large logs: } p_T^b \text{ cut select high } p_T \text{ b's.}
  \]

NLO calculation drastically improves the stability of the cross section.
- More **inclusive cross sections**, two approaches:
  - Fixed (or four) flavor number scheme (FFS):
    fixed order approach, based on $gg, q\bar{q} \rightarrow b\bar{b}H$ only.
  - Variable (or five) flavor number scheme (VFS):
    resum large logs in $b$-quark PDF.

Perturbative series ordered in Leading and SubLeading powers of $\alpha_s \ln(\mu^2_H/m_b^2)$. Need to consider (avoiding double counting)

- $\bar{b}b \rightarrow H$ (known at NNLO) $\rightarrow \alpha_s^2 \ln^2(\mu^2_H/m_b^2)$
- $bg \rightarrow bH$ (known at NLO) $\rightarrow \alpha_s^2 \ln(\mu^2_H/m_b^2)$
- $gg \rightarrow \bar{b}bH$ (need LO, known at NLO) $\rightarrow \alpha_s^2$
Inclusive versus Exclusive modes

- More inclusive cross sections are larger.

\[ \sqrt{s} = 2 \text{ TeV} \]
\[ M_t = 175 \text{ GeV} \]

- More exclusive modes have much smaller background.
- Only exclusive modes probe the $b\bar{b}H$ coupling unambiguously.
Exclusive cross section for $p\bar{p}, pp \rightarrow b\bar{b}H$: 2 b-tags

Use high $p_T$ $b$-quarks to suppress background: need NLO $(q\bar{q})gg \rightarrow b\bar{b}H$

Non trivial dependence on renormalization of $m_b(\mu)$ in bottom quark Yukawa coupling: \[ OS,\overline{MS} \rightarrow \text{renormalization scheme of } y_b = m_b/v \]

Theoretical uncertainty: 15% (scale dep.), 15% (ren. scheme dep.)
1b-tag and 0b-tag modes: comparison between FFS and VFS

(1b-tag and 0b-tag modes: comparison between FFS and VFS)

\[ \sigma(pp \to b\bar{b}h + X) [\text{fb}] \]
\[ \sqrt{s} = 1.96 \text{ TeV} \]
\[ \mu = (2m_b + M_h)/4 \]
\[ p_T^{\text{min}} > 20 \text{ GeV} \]
\[ |\eta_{\text{b\bar{b}}}| < 2 \]

\[ gb/b \to b\bar{b}+h \]
\[ gg \to b\bar{b}+h \]

\[ \sigma(pp \to b\bar{b}h + X) [\text{fb}] \]
\[ \sqrt{s} = 14 \text{ TeV} \]
\[ \mu = (2m_b + M_h)/4 \]
\[ p_T^{\text{min}} > 20 \text{ GeV} \]
\[ |\eta_{\text{b\bar{b}}}| < 2.5 \]

\[ gb/b \to b\bar{b}+h \]
\[ gg \to b\bar{b}+h \]
Overview

QCD predictions for total cross sections to Higgs production processes are under good theoretical control:

\[ \sigma(pp \to h + X) \text{ [pb]} \]

\[ M_h [\text{GeV}] \]

Caution:

- uncertainties only include \( \mu_R/\mu_F \) dependence
- uncertainties from PDF’s are not included (but should improve)